

The Black Economy and Education*

Ann-Sofie Kolm[†] and Birthe Larsen[‡]

February 1, 2010

Abstract

This paper develops an equilibrium search and matching model with informal sector employment opportunities and educational choice. We show that informal sector job opportunities distort educational attainment inducing a too low stock of educated workers. As informal job opportunities to a larger extent face low skilled workers, combating the informal sector improves welfare as it increases the incentives for education. However, too aggressive combating of the informal sector is not optimal as that induces inefficiently high unemployment rates.

JEL-codes: H26, I21, J64

Keywords: Tax evasion, underground economy, education, matching, unemployment.

*We want to thank Gerard van den Berg, Per Engström, Bertil Holmlund, Tomas Lindström, Rick van der Ploeg, Åsa Rosén, participants at a CESifo workshop, a CIM workshop, the conference on heterogenous labour: search friction and human capital investments, and seminars at Uppsala University, Copenhagen Business School, Aarhus School of Business, Stockholm School of Economics, and CEBR. The first version of the paper was written while the first author was visiting the University of Michigan. Their hospitality is greatly acknowledged.

[†]Department of Economics, Stockholm University, S-106 91 Stockholm, Ph. +46 8 163547, Fax +46 8 161425. E-mail address: ann-sofie.kolm@ne.su.se

[‡]Insead, CEBR and Copenhagen Business School, E-mail address: Birthe.LARSEN@insead.edu

1 Introduction

The last decade has witnessed a surge of academic as well as journalistic writings on tax evading activities.¹ One reason for this interest is that tax evasion opportunities may create various distortions in the behaviour of economic agents. For example, too much work may be carried out in the informal sector if consumer wages are relatively higher in this sector.²

In this paper we argue that the choice to acquire higher education may also be distorted by tax evasion opportunities. If workers with a lower level of education to a larger extent face job opportunities in an informal sector, there are fewer incentives to acquire higher education. Policies combating the informal sector could therefore be expected to increase the educational attainment in the economy.

Empirical observations seem consistent with that workers with a lower level of education face relatively more job opportunities in the informal sector. For example, Boeri and Garibaldi (2002) show for Sicily, that mainly workers at the lower end of the skill distribution engage in informal activities. Pedersen and Smith (1998), using comprehensive survey data, show that almost half of the informal sector activities in Denmark are carried out within the construction sector. They also find that around 70 percent of the total hours performed in the informal sector is carried out within the service sector or construction sector. Furthermore, Pedersen (2003), using the same questionnaire design for five countries, concludes that most informal activity in Great Britain, Denmark, Norway, and Sweden is carried out within the construction sector. In Germany most informal activity takes place within a group of sectors denoted: Fishing, Agriculture (including tree-felling and gardening) and Mining. Finally, performing logistic regressions for the five countries, Pedersen (2003) confirms that skilled blue colour workers carry out more black market activities than others, and that the likelihood of black market activities falls with the length of education.³

¹See Slemrod and Yitzhaki (2002) and Schneider and Eneste (2000) for two surveys of tax avoidance and tax evasion.

²See Pedersen and Smith (1998) for evidence for Denmark. One can note that tax evasion opportunities may, in fact, also reduce distortions. Tax evasion may, for example, reduce the distortion caused by too high consumption of untaxed leisure.

³Note also that in the literature on crime, it is often stressed that workers with low market wages have more incentives to commit crimes. Most empirical studies of criminal incomes

The aim of the paper is threefold. *First*, we want to examine how labour market performance and incentives to acquire education is affected by informal employment opportunities. *Second*, we want to study how tax and punishment policies affect wage formation, unemployment, educational attainment, and the size of the informal sector. *Third*, we want to characterize the optimal tax and punishment system.

To that end, we develop a four-sector equilibrium model featuring matching frictions and worker-firm wage bargains. Workers differ in ability and decide whether or not to acquire higher education. Unemployed workers search for jobs in both the formal and the informal sector by allocating their search effort optimally between the sectors.

To keep the model simple, the differences between the formal and the informal sector, as well as the differences between those that acquire higher education and those that do not, are kept at a minimum. The only difference between the formal and the informal sector is that taxes are not paid in the latter. In case a worker firm pair is detected evading taxes, a fine has to be paid and the worker firm pair is dissolved. Highly educated workers differ from workers not acquiring education (from now on referred to as manual workers) as they have higher productivity and face longer expected employment spells. The OECD employment outlooks and other studies verifies that higher educational attainment is associated with both higher productivity and shorter expected unemployment spells.⁴

We find that manual workers face relatively larger informal sector employment opportunities. This is because shorter expected employment spells increase the relative payoff of opening informal sector vacancies. Consequently, higher fines on tax evasion increase the incentives to acquire higher education

find that criminal activities offers low skill men higher hourly wages than legitimate activities. See Freeman (1999) for a survey of the economics of crime. Also, the theoretical study by Burdett, Lagos and Wright (2003) develops a model where workers are less likely to commit crimes when their wages are higher. This has a parallel to informal sector work where high skill workers may not to the same extent get full pay-off for their higher productivity, which makes informal activities less attractive for highly educated workers.

⁴The fact that a higher level of initial educational attainment is associated with longer tenure in the firm at all levels of labour market experiences was documented already in the OECD employment outlook 1993 (chapter 4). The recent study by Gartell et al (2010) using linked employer employee data also shows that job destruction rates decrease with the educational level. For more highly educated workers, fewer jobs are destroyed.

as the attractiveness of informal employment is reduced. Thus, a more severe punishment of the informal sector increases the stock of educated workers.

More severe punishment will also, most likely, improve welfare. This follows as both taxation and the fact that the informal sector is punished to a lesser extent than the formal sector is taxed, implies that too few workers choose to educate themselves. However, it also follows because the unemployment rate is too high, and the number of vacancies in relation to the number of job searchers is too low, from a welfare point of view, when the informal sector is punished to a smaller extent than the formal sector is taxed. One can note that this is the case even if the original Hosios condition is imposed. Considering optimal tax and punishment policy, we find that it is optimal to choose the punishment rates so to more than fully counteract the distortion created by the government's inability to tax the informal sector.

Early theoretical analyses of tax evasion are provided by Allingham and Sandmo (1972) and Srinivasan (1973), where underreporting of income is modeled as a decision made under uncertainty. Subsequent papers have enhanced the basic model of individual behaviour by, for example, incorporating endogenous labour supply decisions.⁵ Also general equilibrium models with tax evasion have been developed (for an example see Cremer and Gahvari (1993)). Several theoretical papers have also recognized that the opportunities for tax evasion differ across occupations. See for example Pestieau and Possen (1991). Occupational choice in this literature is usually thought of as a choice between self-employment, where under-reporting is possible, and regular employment, where under-reporting is not an option.

The principal contribution of the analyses in this paper is that we shed light on how the presence of work opportunities in an informal sector can affect the educational attainment in an economy, and thereby welfare. This has, to our knowledge, not been explored in the previous literature. Moreover, we consider tax evasion in a model featuring involuntary unemployment, which, in contrast to most previous work on tax evasion, enables us to study the impact of tax evasion on wage formation and unemployment. During the last decade there has been focus on how tax and punishment policies affect involuntary unemployment; see Kolm and Larsen (2001,2002), Cavalcanti (2002), Boeri and

⁵See for example Andersen (1977) and Sandmo (1981) for early contributions of endogenous labour supply and underreporting of income.

Garibaldi (2002), and Fugazza and Jacques (2003). These papers however rely on different mechanisms to generate coexistence of formal and informal jobs than what is done in this paper.⁶ There is also a recent paper by Albrecht et al (2009) who considers the impact of payroll taxes and severance pay on unemployment in the presence of an informal sector. Their informal sector is modelled from a Latin American perspective where the informal sector is large and can be seen as an unregulated sector which is not affected by these policies. As their focus is not on the illegality of this sector, punishment policies of informality are not modelled.

2 The model

The economy consists of a labour force which differs in ability to acquire education. Abilities, e , are uniformly distributed between 0 and 1, $e \in [0, 1]$, and based on ability, workers decide whether or not to proceed to higher education.⁷ The cost of higher education, $c(e)$ is decreasing in ability, $c'(e) < 0$.

Workers face job opportunities in both a formal and an informal sector. The economy thus consists of four sectors; the formal and informal sector for manual workers (denoted F, m and I, m), and the formal and informal sector for highly educated workers (denoted F, h and I, h). Highly educated workers differ from manual workers as they have higher productivity and face longer expected employment spells.

2.1 Matching

Both manual and highly educated workers search for jobs in a formal and an informal sector. For simplicity, we assume that only unemployed workers search for jobs. This is a simplification, i.e. we do not acknowledge that the connection to the labour market given by working in the formal sector may bring about job opportunities not available while unemployed. Workers accept job offers

⁶This paper assumed directed search where unemployed workers are optimally allocating search effort into both the formal and the informal sector. This particular modelling strategy of search effort has a close resemblance to how search is modelled in van den Berg and van der Klaauw (2006), where search for a job can be conducted using different search channels.

⁷It is assumed to be costless to become a manual worker, but that workers who get a higher education find it costly to do so. This is a normalisation and has no consequences for the results.

as long as the expected payoff exceeds their reservation wage. The matching functions for the four categories of jobs are given by $X_l^j = \left(v_l^j\right)^{\frac{1}{2}} \left(\left(\sigma_l^j\right)^{\gamma} u_l\right)^{\frac{1}{2}}$, where X_l^j is the sectorial matching rate, v_l^j , is the sectorial vacancy rate, and u_l , is the unemployment rate, $j = F, I$ and $l = m, h$. The rates are defined as the numbers relatively to the labour force of manual and highly educated workers, respectively. The exponents in the matching function is set to be equal to a half. This is done so to simplify the welfare analysis where we derive the optimal tax and punishment system when we have imposed the traditional Hosios condition. In that case we can disregard congestion externalities as the elasticity of the expected duration of a vacancy is equal to the bargaining power of workers in a symmetric Nash bargaining situation.

Workers allocate search effort optimally across the formal and the informal sector. Each worker's total search intensity is exogenously given and normalized to unity, where $\sigma_l^I = \sigma_l$, denotes search effort directed towards the informal sector, and $\sigma_l^F = 1 - \sigma_l$, $l = m, h$, denotes search effort directed towards the formal sector. The parameter $\gamma < 1$ captures that the effectiveness of search falls with search effort, i.e., the first unit of search in one sector is more effective than the subsequent units of search. This could capture that different search methods are used when searching for a job in a market. The more time that is used in order to search in a market, the less efficient search methods have to be used.

The transition rates into informal and formal sector employment for a particular worker i , are $\lambda_{li}^I = \sigma_{li}^{\gamma} \left(\theta_l^I\right)^{\frac{1}{2}}$ and $\lambda_{li}^F = (1 - \sigma_{li})^{\gamma} \left(\theta_l^F\right)^{\frac{1}{2}}$, where $\theta_l^I = \frac{v_l^I}{\sigma_l^{\gamma} u_l}$ and $\theta_l^F = \frac{v_l^F}{(1 - \sigma_l)^{\gamma} u_l}$ are labour market tightness, $l = m, h$, measured in effective search units. The rates at which vacant jobs become filled are $q_l^j = \left(\theta_l^j\right)^{-\frac{1}{2}}$, $j = F, I$, $l = m, h$.

2.2 Value functions

Let U_l , E_l^F , and E_l^I denote the expected present values of unemployment and employment for manual and highly educated workers. The value functions for

worker i then reads:

$$rU_{li} = R + \lambda_{li}^F(E_l^F - U_{li}) + \lambda_{li}^I(E_l^I - U_{li}) - I(l)c(e_i), l = m, h, \quad (1)$$

$$rE_{li}^F = R + w_{li}^F(1 - t) + s_l(U_l - E_{li}^F) - I(l)c(e_i), l = m, h, \quad (2)$$

$$rE_{li}^I = R + w_{li}^I(1 - p\delta) + (s_l + p)(U_l - E_{li}^I) - I(l)c(e_i), l = m, h, \quad (3)$$

where r is the exogenous discount rate and w_l^j is the sector wage. s_l is the exogenous separation rate, where highly educated workers to a smaller extent are separated from their jobs and thus face longer expected employment spells, i.e., $s_h < s_m$. R is a lump sum transfer that all individuals receive from the government which reflects that the government has some positive revenue requirements.⁸ Highly educated workers pay the individual educational costs $c(e_i)$, where e_i is the worker's ability, $e_i \in [0, 1]$, $c'(e_i) < 0$ and $c''(e_i) > 0$. In order to guarantee a non-trivial solution where some, but not all, individuals choose to acquire education, the individual with highest ability face a very low costs of education, more specifically $c(1) = 0$, and the individual with the lowest ability face very high cost of education, i.e., $\lim_{e \rightarrow 0} c(e) = \infty$. The indicator function $I(l)$, $l = m, h$ takes the value zero for manual workers and the value one for highly educated workers, hence $I(m) = 0$ and $I(h) = 1$.⁹ The parameter t is the proportional income tax rate, p is the probability of being detected working in the informal sector, and δ is the proportion of the evaded income the worker has to pay as a punishment fee if detected. The match is dissolved when detected which implies that the separation rate in the informal sector exceeds the formal sector separation rate. For simplicity, we disregard unemployment benefits.

Let J_l^F and V_l^F represent the expected present values of an occupied job and a vacant job in the formal sector, respectively. The arbitrage equations for the formal sector of a job paying the wage w_{li}^F and a vacant job are then

$$rJ_{li}^F = y_l - w_{li}^F(1 + z) + s_l(V_l^F - J_{li}^F), l = m, h, \quad (4)$$

$$rV_l^F = q_l^F(J_l^F - V_l^F) - k, l = m, h, \quad (5)$$

⁸Everyone receives this transfer. The government cannot exclude the informal sector workers as the government does not know who the informal sector workers are (if it did, it could just punish all of them).

⁹We assume that the educational cost is a cost to acquire and maintain skill. This is a simplifying assumption and is not important for the results. The assumption enables us to use a model without having workers continuously being born and dying. Such a model would, however, generate the same qualitative expressions.

where z is the payroll tax rate and y_l is productivity, $y_h \geq y_l$. Vacancy costs are denoted k . Analogous notation for the informal sector yields:

$$rJ_{li}^I = y_l - w_{li}^I(1 + p\alpha) + (s_l + p)(V_l^I - J_{li}^I), l = m, h, \quad (6)$$

$$rV_l^I = q_l^I(J_l^I - V_l^I) - k, l = m, h, \quad (7)$$

where α is the proportion of the evaded wage the firm has to pay as a punishment fee if detected.

The unemployed worker i allocates search between the two sectors, σ_{li} , in order to maximize the value of unemployment, rU_{li} . A necessary condition for an interior solution is that $\gamma < 1$, which holds by assumption. The first order condition can be written as:

$$\frac{(1 - \sigma_{li})^{1-\gamma}}{(\sigma_{li})^{1-\gamma}} = \left(\frac{\theta_l^F}{\theta_l^I} \right)^{\frac{1}{2}} \frac{E_l^F - U_{li}}{E_l^I - U_{li}}, l = m, h. \quad (8)$$

Workers allocate their search between sectors to equalize the marginal returns to search effort across the two sectors.

2.3 Wage determination

When a worker and firm meet they bargain over the wage, w_{li}^j , taking economy wide variables as given. The first order conditions from the Nash bargaining solutions, with the worker's bargaining power being equal to a half, can be written as:

$$\frac{1}{\phi^F} J_l^F = E_l^F - U_l, l = m, h, \quad (9)$$

$$\frac{1}{\phi^I} J_l^I = E_l^I - U_l, l = m, h, \quad (10)$$

where $\phi^F = \frac{1+z}{1-t}$ and $\phi^I = \frac{1+p\alpha}{1-p\delta}$ are the tax and punishment wedges, and where we have imposed symmetry and the free entry condition, $V_l^j = 0$, $j = F, I$, $l = m, h$.

We can now derive an equation determining how search is allocated between the formal and the informal sectors in a symmetric equilibrium by substituting (9) and (10) into (8) and using that $J_l^F = \frac{k}{q_l^F}$ and $J_l^I = \frac{k}{q_l^I}$ from (5) and (7) together with free entry. This yields:

$$\frac{(1 - \sigma_l)^{1-\gamma}}{(\sigma_l)^{1-\gamma}} = \frac{\theta_l^F}{\theta_l^I} \psi, \quad (11)$$

where

$$\psi = \frac{\phi^I}{\phi^F} = \frac{1 + p\alpha}{1 - p\delta} / \frac{1 + z}{1 - t}, \quad (12)$$

is the wedge between the informal sector and the formal sector. We can interpret a $\psi < 1$ as if the informal sector is punished to a lesser extent than the formal sector is taxed.¹⁰

Equation (11) is a core equilibrium equation. Recall from (8) that workers allocate their search between sectors so that the marginal returns to search effort in the two sectors are equal. With wages being endogenously determined in equilibrium, this implies to account for the wedge, ψ , and for the formal relative to the informal sectorial tightness, θ_l^F / θ_l^I , when deciding how to allocate search. For example, if the informal sector is punished to a smaller extent than the formal sector is taxed, $\psi < 1$, unemployed workers tend to direct more search into the informal sector. And the relatively tighter the formal sector is, the larger formal sector search tends to be.

By use of equation (1)-(7) and (11) in equations (9) and (10), equilibrium producer wages, ω_l^j , are given by:

$$\omega_l^F = w_l^F (1 + z) = \frac{1}{2} \left(y_l + k \frac{\theta_l^F}{(1 - \sigma_l)^{1-\gamma}} \right), l = m, h, \quad (13)$$

$$\omega_l^I = w_l^I (1 + p\alpha) = \frac{1}{2} \left(y_l + k \frac{\theta_l^I}{\sigma_l^{1-\gamma}} \right), l = m, h. \quad (14)$$

Wages increase with labour market tightness and decrease with search intensity in each sector. This follows as a higher labour market tightness and a lower search intensity improve the worker's bargaining position. An increase in tightness makes it easier for an unemployed worker to find a job, and at the same time harder for a firm to fill a vacancy. This improves the worker's relative bargaining position, resulting in higher wage demands. An increase in search, will instead reduce the worker's bargaining position. This follows as the effectiveness of search falls with more search.¹¹

¹⁰In contrast, if $\psi = 1$, the informal sector is punished equally hard as the formal sector is taxed. With risk neutral individuals there is, in one sense, no substantial difference between the tax system and the punishment system since the punishment system is a randomized tax system.

¹¹An alternative way of describing the intuition for why increased sector search reduces

From (13), (14) together with (11) it follows that producer wages in the formal sector exceed informal sector producer wages when $\psi < 1$, and vice versa when $\psi > 1$. Moreover, rewriting (13) and (14) in terms of consumer wages we have that consumer wages in the formal sector subseed informal sector consumer wages when $\psi < 1$, and vice versa when $\psi > 1$. More specifically we have $\omega_l^F - \omega_l^I = \frac{k\theta_l^F}{2(1-\sigma_l)^{1-\gamma}} (1 - \psi)$ and $w_l^F (1 - t) - w_l^I (1 - p\delta) = \frac{y_l}{2\phi^I} (\psi - 1)$.

2.4 Labour market tightness

Labour market tightness for the formal sector and the informal sector are determined by equation (4),(5), (6) and (7) using the free entry condition and the wage equations (13) and (14):

$$k(r + s_l) \left(\theta_l^F \right)^{\frac{1}{2}} = \frac{1}{2} \left(y_l - \frac{k\theta_l^F}{(1 - \sigma_l)^{1-\gamma}} \right), l = m, h, \quad (15)$$

$$k(r + s_l + p) \left(\theta_l^I \right)^{\frac{1}{2}} = \frac{1}{2} \left(y_l - \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right), l = m, h, \quad (16)$$

When $\psi < 1$, and thus the informal sector is punished to a smaller extent than the formal sector is taxed, is the most realistic case. In this case, informal producer wages are lower than formal producer wages and hence the expected instantaneous profits in the informal sector exceed the instantaneous profits in the formal sector. The impact from the wedge smaller than one therefore makes it more attractive for firms to enter the informal sector which tends to make informal tightness exceed formal tightness, that is tends to raise θ_l^I relatively to θ_l^F . However, as the separation rate in the formal sector is lower, $s_l < s_l + p$, the formal sector expected job duration is longer. This makes the formal sector more attractive to enter, tending to raise θ_l^F relatively to θ_l^I .

As from real world observations we have that the formal sector is larger than the informal sector, we here focus on the relevant case when $\theta_l^F > \theta_l^I$. Thus, we assume that the discouraging effect on firms entering the informal sector due to that informal jobs are more insecure, dominates the positive effect on firms

wage demands is the following. When search time into one sector increases, the bargaining position for firms in that sector improves. This follows as firms in that sector then will find it easier to match with a new worker in case of no agreement. The improved bargaining position for firms moderates wage pressure. Analogous reasoning holds in case search effort into a sector falls.

entering the informal sector due to lower expected punishment in relation to taxation.

2.5 Education

When workers decide whether to acquire higher education or remain as a manual worker, they compare the value of unemployment as an educated worker to the value of unemployment as a manual worker. Workers with low ability find it too costly in terms of effort to acquire higher education, whereas high ability workers find it more than worthwhile to do so since they face lower costs of education. The marginal worker has an ability level, \hat{e} , which makes him just indifferent between acquiring higher education and remaining as a manual worker. We write the condition determining the ability level of the marginal worker as:

$$rU_h = rU_m. \quad (17)$$

We can rewrite this expression in a number of ways using earlier equations. For example, by using equations (1)-(3), it is clear that the value of unemployment for a worker is captured by $rU_l = R + \mu_l^F w_l^F (1 - t) + \mu_l^I w_l^I (1 - p\delta) - I(l) c(e)$, $l = m, h$. This expression reveals that it is the consumer wages in the formal and informal sector, weighted by the employment opportunities in each sector, as well as the cost of education, that is important for the educational choice.¹²

As wages are endogenous, we can use equations (1) and (17) together with the first order conditions for wages, and equations (5), (7), (11), together with the free entry condition which gives the following condition:

$$c(\hat{e}) = \frac{k}{\phi^F} \left(\frac{\theta_h^F}{(1 - \sigma_h)^{1-\gamma}} - \frac{\theta_m^F}{(1 - \sigma_m)^{1-\gamma}} \right). \quad (18)$$

Equation (18) gives \hat{e} as a function of the endogenous variables θ_l^F and σ_l , $l = m, h$. Workers with $e \leq \hat{e}$, choose not to acquire education, whereas workers with $e > \hat{e}$ acquire education. Hence, \hat{e} and $1 - \hat{e}$ constitute the manual and educated labour forces, respectively.

If either the expected employment spells are larger for educated worker than manual workers, $s_m > s_h$, or highly educated workers are more productive than

¹²The weights, $\mu_l^F = \lambda_l^F / (r + s_l + \lambda_l^F + \frac{r+s_l}{r+s_l+p} \lambda_l^I)$ and $\mu_l^I = \lambda_l^I / (r + s_l + p + \lambda_l^I + \frac{r+s_l+p}{r+s_l} \lambda_l^F)$, reduce down to $\mu_l^F = n_l^F$ and $\mu_l^I = n_l^I$ when the discount rate approaches zero.

manual workers, $y_h > y_m$ the expected income when becoming an educated worker exceeds the expected income of remaining as a manual worker, i.e., the right hand side in (18) is positive. This induces that at least some high ability individuals will choose to educate themselves as $c(1) = 0$. Moreover, as individuals with very low ability face very high costs of education, i.e., $\lim_{e \rightarrow 0} c(e) = \infty$, a non-trivial interior solution for \hat{e} is guaranteed. See the appendix for the proof of existence of $\hat{e} \in (0, 1)$.

2.6 Unemployment

The equations determining the employment rates in the formal sector and the informal sector, n_l^F, n_l^I , and the unemployment rates, $u_l, l = m, h$, are given by the flow equilibrium equations and the labour force identity.¹³ The official unemployment rate u_l^o , is given by $u_l^o = u_l + n_l^I$. Solving for the employment and unemployment rates yield:

$$n_l^I = \frac{\frac{\lambda_l^I}{s_l + p}}{1 + \frac{\lambda_l^I}{s_l + p} + \frac{\lambda_l^F}{s_l}}, n_l^F = \frac{\frac{\lambda_l^F}{s_l}}{1 + \frac{\lambda_l^I}{s_l + p} + \frac{\lambda_l^F}{s_l}}, l = m, h, \quad (19)$$

$$u_l = \frac{1}{1 + \frac{\lambda_l^I}{s_l + p} + \frac{\lambda_l^F}{s_l}}, u_l^o = \frac{1 + \frac{\lambda_l^I}{s_l + p}}{1 + \frac{\lambda_l^I}{s_l + p} + \frac{\lambda_l^F}{s_l}}, l = m, h. \quad (20)$$

The actual and official total number of unemployed workers are given by $U_{TOT} = \hat{e}u_m + (1 - \hat{e})u_h$, $U_{TOT}^o = \hat{e}u_m^o + (1 - \hat{e})u_h$.

2.7 Manual versus highly educated workers

Comparing unemployment and the size of the informal sector in relation to the formal sector, we can conclude the following.

Proposition 1 *The relatively longer expected employment spells for highly educated workers, i.e. $s_m > s_h$, implies that manual workers face a higher unemployment rate, i.e., $u_m > u_h$, and a larger informal relative to formal sector, i.e., $n_m^I/n_m^F > n_h^I/n_h^F$.*

¹³For highly educated workers $\lambda_h^I u_h \hat{e} = (s_h + p) n_h^I \hat{e}$, $\lambda_h^F u_h \hat{e} = s_h n_h^F \hat{e}$, and $n_h^F + n_h^I = 1 - u_h$, and for manual workers $\lambda_m^I u_m (1 - \hat{e}) = (s_m + p) n_m^I (1 - \hat{e})$, $\lambda_m^F u_m (1 - \hat{e}) = s_m n_m^F (1 - \hat{e})$, and $n_m^F + n_m^I = 1 - u_m$

Proof. See appendix. ■

The expected job duration for manual workers being shorter than for highly educated workers, tends to increase the unemployment rate facing manual workers. However, the higher unemployment rate for manual workers is also due to the fact that a higher separation rate facing those workers makes it less attractive to open vacancies. Therefore labour market tightness facing manual workers is relatively lower in both the formal and the informal sector.

The relative size of the informal sector is also affected by the separation rate. A higher separation rate hits the relatively larger formal sector more severely inducing formal sector tightness to fall by more than informal sector tightness. Thus, as the expected job duration for manual workers are shorter than for highly educated workers, manual workers will face relatively more job opportunities in the informal sector. More job opportunities in the informal sector for manual workers imply that they search for more jobs in the informal sector in relation to the formal sector than highly educated workers do. As a consequence, the relative size of the informal sector is larger for manual workers than for highly educated workers.

3 Comparative statics

This section is concerned with the impact of the tax and punishment system on labour market performance. Proofs of all Propositions follow from straightforward comparative statics and are available upon request.

We only consider fully financed changes in the punishment rates. Hence, changes in the punishment rates, α or δ , are always followed by adjustments in the tax rates, z or t , so as to balance the government budget restriction. The government budget restriction is given by:

$$\hat{e} \sum_{j=F,I} \left(n_m^j \omega_m^j \left(1 - \frac{1}{\phi^j} \right) \right) + (1 - \hat{e}) \sum_{j=F,I} \left(n_h^j \omega_h^j \left(1 - \frac{1}{\phi^j} \right) \right) - \xi(p) = R, \quad (21)$$

where R is the exogenous revenue requirements and $\xi(p)$ is auditing costs.¹⁴

¹⁴From (21) it follows that any R can be attained by increasing ϕ^F and ϕ^I simultaneously so to keep ψ constant. Also, as only ψ affects the variables $\sigma_l, \theta_l^F, \theta_l^I, n_l^F, n_l^I, u_l, u_l^o, l = m, h$, the effects of fully finance reforms on these variables can be considered by only looking at changes in ψ . Thus the government budget restriction can be ignored in section 3.1 and 3.2.

3.1 Tightness and search

The effects on tightness and the allocation of search across the formal and the informal sector are summarized in the following proposition.

Proposition 2 *A fully financed increase in the punishment rate (δ or α) will reallocate search intensity towards the formal sector (σ_l falls). Furthermore it will increase tightness in the formal sector (θ_l^F) and reduce tightness in the informal sector (θ_l^I).*

When tax evasion is punished more severely, unemployed workers will find it optimal to reallocate their search towards the formal sector. However, when search effort is reallocated towards the formal sector, wage demands in the formal sector fall whereas wage demands in the informal sector increase. This follows as the effectiveness of search in the formal sector falls, which weakens the workers' bargaining position. In contrast, the reduced search in the informal sector increases the effectiveness of informal sector search, which improves the bargaining position for workers in the informal sector. The relatively lower producer wages in the formal sector induce firms to exit the informal sector and enter the formal sector; formal sector tightness increases whereas informal sector tightness falls.

3.2 Employment and unemployment rates

A fully financed increase in the punishment of the informal sector causes the formal sector employment rate, (n_l^F) to increase as more workers transit into formal employment. This, in turn, is caused by the increase in formal sector tightness and the reallocation of search effort towards the formal sector. In contrast, as tightness and search effort in the informal sector fall, the transition rate into informal employment, and consequently the sectorial employment rate, (n_l^I) falls.

The sectorial reallocation of employment affects the unemployment rate. We have the following results:

Proposition 3 *A fully financed increase in the punishment rate (δ or α) will cause the official unemployment rate (u_l^o) to fall unambiguously and the actual unemployment rate (u_l) to fall if $\psi < 1$, $l = m, h$.*

The unemployment rate falls as workers are reallocated towards the formal sector where jobs last on average a longer time.¹⁵ In addition, the official unemployment rate falls both because the actual unemployment rate falls, and because workers are reallocated towards the formal sector.

3.3 Education

A closer examination of (18) reveals that changes in the punishment rates, $\phi^I = \frac{1+p\alpha}{1-p\delta}$, affects the share of educated workers, $1 - \hat{e}$, through ψ only, whereas changes in the tax rates, $\phi^F = \frac{1+z}{1-t}$, have a direct effect on $1 - \hat{e}$ in addition to the effects working through ψ . Therefore, in order to consider the effects of a fully financed change in the punishment rates on the number of educated workers, we have to account for repercussions on $1 - \hat{e}$ following adjustments in the tax rates. However, let us first consider the impact on $1 - \hat{e}$ of a change in the tax and punishment rates separately:

$$\frac{\partial(1 - \hat{e})}{\partial\phi^I}|_{\phi^F} = -\frac{k}{\phi^F} \frac{(1 - \gamma)}{\phi^I c'(\hat{e})} \left(y_h A - \frac{k\theta_h^F}{(1 - \sigma_h)^{1-\gamma}} B \right) > 0, \quad (22)$$

$$\frac{\partial(1 - \hat{e})}{\partial\phi^F}|_{\phi^I} = -\psi \frac{\partial(1 - \hat{e})}{\partial\phi^I}|_{\phi^F} + \frac{c(\hat{e})}{c'(\hat{e})\phi^F} < 0. \quad (23)$$

where $A = \frac{(1 - \sigma_m)^{\gamma-2}}{\theta_m^I} \frac{y_m^2}{-D_m} - \frac{(1 - \sigma_h)^{\gamma-2}}{\theta_h^I} \frac{y_h}{-D_h}$ and $B = \frac{(1 - \sigma_m)^{\gamma-2}}{\theta_m^I} \frac{y_m}{-D_m} - \frac{(1 - \sigma_h)^{\gamma-2}}{\theta_h^I} \frac{y_h}{-D_h}$, $D_l = \frac{1}{\theta_l^F \theta_l^I} y_l \frac{(1 - \gamma)}{1 - \sigma_l} \frac{1}{\sigma_l} \left(y_l - k \frac{\theta_l^F}{(1 - \sigma_l)^{1-\gamma}} \psi \left(1 + \sigma_l \left(\frac{1}{\psi} - 1 \right) \right) \right) < 0$ and $A \geq B$ as $y_m \leq y_h$ and irrespective of the size of ψ .

Equation (22) shows that the number of educated workers increases with higher punishment rates for a given tax system. This summarizes the impact through several channels. When manual workers face more informal vacancies relative to formal vacancies than the highly educated workers do, i.e., $\theta_m^I/\theta_m^F > \theta_h^I/\theta_h^F$ manual workers will also to a relatively larger extent transit into informal sector jobs than highly educated workers. This follows both because more informal vacancies are available relative to formal vacancies for manual workers, but also because manual workers will search for jobs in the

¹⁵In case $\psi > 1$, the number of vacancies relatively to unemployment is reduced which will have a counteracting effect on unemployment. Consequently, the unemployment rate may increase in this case. Recall that $\psi < 1$ corresponds to the empirically most plausible case where producer wages in the formal sector exceed those in the informal sector, and where consumer wages in the informal sector exceeds those in the formal sector.

informal sector to a larger extent than highly educated workers, i.e., $\sigma_m > \sigma_h$ when $\theta_m^I/\theta_m^F > \theta_h^I/\theta_h^F$. Thus, when the punishment rate increases, manual workers are hit more severely by the reduced attractiveness of the informal sector than highly educated worker.

Equation (23) gives the impact on the number of educated workers as the tax rates increase, for a given punishment system. The *first* term captures the effect on the educated labour force of an increase in the tax rates working through ψ . Increased taxation induces the opposite movements as was described in connection to equation (22) and analogous reasoning can be conducted. The *second* term in (23) captures the impact of increased taxation for a given wedge, ψ . Increased taxation reduces the consumer wages for both educated and manual workers, but it will not affect the costs of higher education. Consequently, as the value of being an educated worker falls by relative more than the value of being a manual worker, the number of educated workers falls. As the effects work in the same direction, we can conclude that the number of educated workers in the economy tends to decrease with increased taxation.

In order to consider fully financed increases in the punishment rates, we need to consider the impact of the tax and punishment rates on the government revenues. The tax and punishment rates affect the government revenues in a number of ways (details are given in an appendix available upon request). However, assuming that we are located on the positively sloped side of the Laffer curves, and hence dynamic adjustments in equilibrium wages, employment rates and labour forces are not dominating the direct effects, the government revenue increases with increased tax and punishment rates. An increase in the punishment rate accordingly calls for reductions in the tax rates in order to maintain a balanced budget. Considering that we are located on the positively sloped side of the Laffer curve, we have the following result:

Proposition 4 *A fully financed increase in the punishment rate (δ or α) will increase the number of educated workers.*

As manual workers to a larger extent face job opportunities in an informal sector, there are less incentives to acquire higher education. However, when the punishment of informal work increases, the attractiveness of remaining as a manual worker with access to many informal work opportunities is reduced, and consequently more workers will choose to proceed to higher education.

3.4 Unemployment

This section is concerned with how the number of unemployed workers is affected by changes in the tax and punishment systems. As is clear from section 2.6, the total number of unemployed workers, as well as the number of officially unemployed workers, depends on the division of labour across sectors. Since the division of labour across sectors depends on the tax rates, ϕ^F , separate from the wedge, ψ , we have to account for the government budget restriction explicitly. Again we assume that we are located on the positively sloped side of the Laffer curves.

Proposition 5 *A sufficient condition for a fully financed increase in the punishment rate (δ or α) to reduce total unemployment, $\frac{\partial U_{TOT}}{\partial \phi^F} < 0$, is that $\psi \leq 1$.*

Total unemployment falls because the unemployment rate for both manual and educated workers is reduced as workers are reallocated towards the formal sector when $\psi < 1$, as well as because more workers choose to educate themselves (as $u_h < u_m$).

4 Welfare

This section is concerned with welfare analysis and the optimal design of tax and punishment policies. As shown, punishing the informal sector increases the number of educated workers as it reduces the relative attractiveness of being a manual worker. This is essential when considering the impact on welfare. In case the educated labour force is inefficiently low, this provides an additional argument for fighting the informal sector. If, on the other hand, the educated labour force is inefficiently high, combating the informal sector induces a welfare cost as the educated labour force then becomes even higher.

This suggests that the punishment rates are potential instruments that can be used to affect the number of educated workers and thereby welfare. Even though there are other, potentially more direct, instruments to correct for inefficiencies in the educational level, it is of importance to note that the wedge actually has an impact on the number of educated workers and thereby potentially an impact on welfare in the economy.

We make use of a utilitarian welfare function, which is obtained by adding all individuals' and firms' steady state flow values of welfare. This accounts for

that both the formal and the informal economy generate welfare in the economy. The social welfare function is written as:

$$W = \hat{e}\tilde{W}_m + \int_{\hat{e}}^1 \tilde{W}_h de,$$

where

$$\begin{aligned}\tilde{W}_m &= u_m r U_m + n_m^F r E_m^F + n_m^I r E_m^I + n_m^F r J_m^F + n_m^I r J_m^I + v_m^F r V_m^F + v_m^I r V_m^I, \\ \tilde{W}_h &= u_h r U_h + n_h^F r E_h^F + n_h^I r E_h^I + n_h^F r J_h^F + n_h^I r J_h^I + v_h^F r V_h^F + v_h^I r V_h^I\end{aligned}$$

We assume that firms are owned by "renters" who do not work. By making use of the asset equations for workers and firms in the three sectors, imposing the flow equilibrium conditions as well as the government budget restriction in (21), and considering the case of no discounting, i.e., $r \rightarrow 0$, we can write the welfare function as:

$$W = \hat{e}W_m + \int_{\hat{e}}^1 W_h de - \xi(p), \quad (24)$$

where

$$W_m = (1 - u_m) y_m - u_m \Theta_m k, \quad (25)$$

$$W_h = (1 - u_h) y_h - u_h \Theta_h k - c(e), \quad (26)$$

where $\Theta_l = \left(\theta_l^I (\sigma_l)^\gamma + \theta_l^F (1 - \sigma_l)^\gamma \right)$, $l = m, h$.¹⁶ With the assumption of risk neutral individuals, we ignore distributional issues and hence wages will not feature in the welfare function.

Examining the welfare measure in (24) reveals that we have to consider both how the tax and punishment system affects the welfare of the manual and highly educated sector, W_m and W_h , and how it affects the stock of highly educated workers, $1 - \hat{e}$. We see from the welfare specification in (24), (25), and (26) that changes in the punishment rates, δ or α , only affects welfare through its effect on the wedge, ψ . The wedge, in turn, affects total welfare, W , through its impact on welfare for manual and highly educated workers, W_m and W_h , and through its impact on the number workers acquiring education, $1 - \hat{e}$. Changes in the tax rates, t and z , on the other hand, affects W through the wedge, ψ , but it will also have a direct effect on the number of workers acquiring education.

¹⁶This welfare measure is analogous to the welfare measure described in, for example, Pissarides (2000) as it includes the aggregate production minus total vacancy costs, i.e. we note that $u_l \Theta_l k = (v_l^F + v_l^I) k$, $l = m, h$.

Let us first consider how a change in the number of workers acquiring education influences the social welfare measure:

$$\frac{\partial W}{\partial (1 - \hat{e})} = W_h(\hat{e}) - W_m = n_h^F \omega_h^F + n_h^I \omega_h^I - c(\hat{e}) - n_m^F \omega_m^F - n_m^I \omega_m^I. \quad (27)$$

Welfare increases when more workers acquire education whenever the number of educated workers is too low from a welfare point of view. Similarly, welfare falls as more workers acquire education when too many workers are educated from a welfare perspective.¹⁷

By looking at the market solution for the stock of educated workers, it is clear that it is exclusively the tax and punishment system that distorts educational choice. The equation for educational choice (18) can be written as $\frac{1}{\phi^F} \left(n_h^F \omega_h^F + \frac{1}{\psi} n_h^I \omega_h^I - n_m^F \omega_m^F - \frac{1}{\psi} n_m^I \omega_m^I \right) = c(\hat{e})$ by using the expressions for rU_l presented above equation (18). We note that there are two sources through which the tax and punishment system distorts the educational choice, namely through the punishment tax wedge, ψ , and through the tax rates, ϕ^F .

The *first* distortion on educational choice appears when ψ departs from unity. When $\psi < 1$, being a manual worker is rather pleasant as relatively more employment opportunities in the attractive informal sector are available. This reduces the incentives of education, which tends to imply that too few workers educate themselves. For $\psi > 1$, on the other hand, employment opportunities in an informal sector are not very attractive as the expected punishment rates are higher than the tax rates. This increases the incentives of education, tending to make too many people educate themselves.

The *second* distortion on educational choice follows as tax rates are positive, $\phi^F > 1$. Taxation hits educated workers more severely than manual workers, which reduces the incentives of education. Hence, positive tax rates distort the educational choice such that too few workers tend to educate themselves.

From this we can conclude that welfare always increases when more workers educate themselves when $\psi \leq 1$. When ψ is smaller than unity there is an inefficiently low stock of educated workers both because positive tax rates hit educated workers more severely and because manual workers face relatively more attractive informal sector employment opportunities. When $\psi = 1$, the fact that tax rates are positive, implies that it is still welfare improving that more workers

¹⁷This clearly follows by definition as W is concave in $(1 - \hat{e})$, and reaches its maximum when $n_h^F \omega_h^F + n_h^I \omega_h^I - c(\hat{e}) - n_m^F \omega_m^F - n_m^I \omega_m^I = 0$.

educate themselves. When $\psi > 1$, the punishment of the informal sector works as an instrument to counteract the distortion created by the tax rates. Further increases in ψ above unity eventually fully counteract the distortion of the tax system, making the market solution stock of educated workers being socially optimal.

Let us now consider the optimal design of tax and punishment systems. Since welfare depends on the division of labour across sectors, and the labour division depends on the tax rates separate from the wedge, we choose the punishment rates to maximize welfare accounting for that the tax rates adjust to balance the government budget restriction. We use (21) to write the tax rates as a function of the punishment rates as $\phi^F = h(\phi^I)$, where $\frac{\partial \phi^F}{\partial \phi^I} < 0$ if we assume that we are located on the positively sloped side of the Laffer curves.

Maximizing W in equation (24), (25), and (26) with respect to ϕ^I enables us to write the first order condition as:

$$\frac{\partial W}{\partial \phi^I} = \frac{\partial W}{\partial (1-\hat{e})} \left(\frac{\partial (1-\hat{e})}{\partial \phi^I} \Big|_{\phi^F} + \frac{\partial (1-\hat{e})}{\partial \phi^F} \Big|_{\phi^I} \frac{\partial \phi^F}{\partial \phi^I} \right) + \left(\frac{\partial W_m}{\partial \psi} + \frac{\partial W_h}{\partial \psi} \right) \left(\frac{\partial \psi}{\partial \phi^I} \Big|_{\phi^F} + \frac{\partial \psi}{\partial \phi^F} \Big|_{\phi^I} \frac{\partial \phi^F}{\partial \phi^I} \right) = 0 \quad (28)$$

We can conclude that:

Proposition 6 *Welfare is maximized when the punishment rate is chosen such that the informal sector is punished to a larger extent than the formal sector is taxed, i.e., $\psi^* > 1$.*

A detailed proof is available upon request. In case the government has a positive revenue requirement, $R > 0$, tax rates have to be positive, and the optimal wedge is larger than unity.¹⁸

The intuition for choosing a punishment wedge larger than unity, as proposed by proposition 6, follows from that the educational stock is inefficiently low in case $\psi \leq 1$. The first term in (28) captures how welfare is affected when more workers acquire education. This term is positive as long as $\psi \leq 1$ (see (27) and proposition (4)). Hence as more educational attainment increases welfare when $\psi \leq 1$, it tends to be welfare improving to increase the punishment wedge above unity to create incentives for education. However, there is an additional source of distortion in the model in case ψ deviates from unity. This distortion is captured

¹⁸A zero tax rate also implies that there are no informal sector that can be punished. Hence it is not possible to obtain the revenue requirements, R unless $\phi^F > 1$.

by the second term in (28). This term captures how the value of being a manual and a highly educated worker is affected by ψ . When ψ deviates from unity there is a distortion between the informal and the formal sector. If $\psi < 1$, too much work from a welfare point of view is carried out in the informal sector, and vice versa when $\psi > 1$.¹⁹ Hence, when $\psi < 1$, welfare improves when increasing ψ both because it increases the stock of educated workers and because it reduces the distortion between the informal and formal sector. However, when $\psi > 1$, additional punishment of the informal sector brings about a negative effect on welfare as it increases the distortion between the formal and the informal sector. Eventually, further increases in the punishment rates will induce welfare to fall.

From proposition (6), we can conclude that:

Corollary 7 *The stock of educated workers are below what is socially optimal when the punishment rate is chosen so to maximize welfare, i.e., when $\psi = \psi^*$.*

This follows from the first order condition determining the optimal punishment rate (28) and equation (27), where the socially optimal number of educated workers are determined by $\frac{\partial W}{\partial(1-\epsilon)} = 0$. From (28) we observe that the welfare maximizing punishment rate is set so that $\frac{\partial W}{\partial(1-\epsilon)} > 0$, fulfilling the first order condition as the second term is negative for $\psi > 1$, which implies that the educated stock is below what is socially optimal. This intuitively follows as it is not optimal to set the punishment so to fully correct for the inefficiently low stock of educated workers, as such a choice implies a distortion between the formal and the informal sector. As it is a trade-off between the two distortions, it is never optimal to fully eliminate one of them.

The results that the punishment rates should be set such that the informal sector is punished to a larger extent than the formal sector is taxed, i.e., $\psi > 1$, should clearly not be taken literally. Merely it points at that it is potentially important for welfare to acknowledge that employment opportunities in an informal sector may reduce the incentive to acquire education.

¹⁹This is further made clear by noting that vacancies per unemployed worker are maximized, and the unemployment rates for manual and highly educated workers, are minimized when $\psi = 1$.

5 Conclusion

This paper endogenized the relative size of the formal and informal sector for highly educated and manual workers. As highly educated jobs were associated with lower job separations than manual sector jobs, manual workers faced higher unemployment and relatively larger job opportunities in an informal sector. These relatively more extensive job opportunities in the informal sector for manual workers implies that the choice of *not* acquiring higher education became more attractive. Hence, informal sector employment opportunities may reduce the incentives to acquire higher education and thereby reduce the educated labour force.

This paper developed a four-sector general equilibrium model featuring matching frictions and worker-firm wage bargains. Workers differed with respect to ability, and the choice of education was endogenously determined. We asked if increased punishment of the informal sector and/or reduced taxation induced more workers to educate themselves. The answer to that question was yes, and the story just told provides the intuition behind this result.

We also studied how unemployment, was affected by increased punishment of the informal sector and/or reduced taxation. Although some recent studies have shed light on how punishment policies affects unemployment, this paper uses a framework which enabled us to study the impact of tax and punishment policies when workers are searching in both a formal and an informal sector. We found that increased punishment most likely would reduce the number of unemployed workers.

Finally, the paper characterized the optimal tax and punishment system. We showed that it was optimal to punish the informal sector to a larger extent than the formal sector is taxed. The optimal choice of tax and punishment system, however, implied an inefficiently low stock of educated workers. The results show that it is potentially important to take into account the impact of the employment opportunities in the informal sector on education, as it in turn, may have consequences for welfare.

References

- [1] Allingham M.G & A Sandmo (1972) "Income tax evasion: A theoretical analysis" *Journal of Public Economics* 1, p 323-338.
- [2] Albrecht J, Navarro L & S Vroman (2009) "The effects of labor market policies in an economy with an informal sector" *Economic Journal* 119, p 1105-1129.
- [3] Boeri T & P Garibaldi (2002) "Shadow activity and unemployment in a depressed labour market" CEPR Discussion paper, DP3433.
- [4] Burdett K, R Lagos, & R Wright (2003) "Crime, inequality, and unemployment", *American Economic Review* 93, 1764-1777.
- [5] Cavalcanti T (2002) "Labor market policies and informal markets" Working Paper, Universidade Nova de Lisboa.
- [6] Cremer H & F Gahvari (1993) "Tax evasion and optimal commodity taxation" *Journal of Public Economics* 50, p 261-275.
- [7] Fugazza M & J-F Jacques (2004) "Labour market institutions, taxation and the underground economy", *Journal of Public Economics* 88, 395-418.
- [8] Freeman R (1999) "The economics of crime" in O Ashenfelter & R Layard (eds.), *Handbook of Labor Economics*, Volume 3C, North-Holland, Amsterdam.
- [9] Gartell M, Jans A-C & H Persson (2010), "The importance of education for the reallocation of labor: Evidence from Swedish linked Employer-Employee data 1986-2002", *Labour Economics* 17, p 206-214.
- [10] Gottschalck, A. O 2006 "Dynamics of Economic Well-Being: Spells of Unemployment 2001–2003" Current Population Reports. Household Economic Studies.
- [11] Kolm A-S & B Larsen (2006) "Wages, unemployment, and the underground economy" *Tax Policy and Labor Market Performance* , MIT press, edited by Jonas Agell and Peter Birch Sørensen.
- [12] Kolm A-S & B Larsen (2003) "Social norm, the informal sector and unemployment" *FinanzArchiv* 59, p 407-427.

- [13] OECD Employment Outlook (1993).
- [14] Pedersen S & N Smith (1998) "Sort arbejde og sort løn i Danmark" (Black Activities and Black Wages in Denmark). *National Økonomisk Tidsskrift*, no. 136, pp. 289-314.
- [15] Pedersen, Søren (2003) "The Shadow Economy in Germany, Great Britain and Scandinavia - a measurement based on questionnaire surveys. *The Rockwool Foundation Research Unit*, Study no 10.
- [16] Pestieau P & Posse U.M (1991) "Tax evasion and occupational choice" *Journal of Public Economics* 45, p 107-125.
- [17] Pissarides C (2000) *Equilibrium search theory*, Cambridge, MIT Press.
- [18] Sandmo A (1981) "Income tax evasion, labour supply, and the equity-efficiency trade-off" *Journal of Public Economics* 16, p 265-88.
- [19] Schneider F & D Eneste (2000) "Shadow economies: size, causes, and consequences" *Journal of Economic Literature*, 38, p 77-114.
- [20] Slemrod J & S Yitzhaki (2002) "Tax avoidance, evasion, and administration" in A.J Auerbach & F Feldstein (eds.) *Handbook of Public Economics* Volume 3, North-Holland, Amsterdam.
- [21] Srinivasan T.N (1973) "Tax evasion, A model" *Journal of Public Economics* 2, p 339-346.
- [22] Van den Berg G.J & B van der Klaauw (2006) "Counselling and monitoring of unemployed workers: Theory and evidence from a controlled social experiment, *International Economic Review* 47, 895-936.

6 Appendix

6.1 Existence of $\hat{e} \in (0, 1)$.

Consider equation (18). High productivity workers face a lower separation rate and identical or higher productivity than manual workers, that is, $s_m > s_h$ and $y_h \geq y_m$. To ensure that $\frac{\theta_h^F}{(1-\sigma_h)^{1-\gamma}} > \frac{\theta_m^F}{(1-\sigma_m)^{1-\gamma}}$ we need to show that,

$\frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}$, decreases in the separation rate, $d\frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}/ds_l < 0$ and falls or is unaffected by an increase in productivity $d\frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}/dy_l \geq 0$.

Differentiating $\frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}$ with respect to s_l gives, after using that $\left(\frac{1-\sigma_l}{\sigma_l}\right)^{1-\gamma} = \frac{\theta_l^F}{\theta_l^I} \psi$:

$$\begin{aligned} \frac{d\frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}}{ds_l} &= \frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \left(\frac{1}{\theta_l^F} \frac{d\theta_l^F}{ds_l} + \frac{1-\gamma}{1-\sigma} \frac{d\sigma_l}{ds_l} \right) \\ &= \frac{-\frac{(1-\gamma)}{(1-\sigma_l)^{1-\gamma}} \frac{2k}{\theta_l^I}}{(1-\sigma_l)(-D_l)} \left(\left(\theta_l^F\right)^{\frac{1}{2}} \left(y_l - \frac{k\theta_l^I}{\sigma_l^{1-\gamma}}\right) \left(\frac{1}{\sigma_l} - 1\right) + \left(\theta_l^I\right)^{\frac{1}{2}} \left(y_l - \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}}\right) \right) \end{aligned}$$

where

$$D_l = -\frac{1}{\theta_l^F \theta_l^I} y_l \frac{(1-\gamma)}{1-\sigma_l} \frac{1}{\sigma_l} \left(y_l - k \frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \psi \left(1 + \sigma_l \left(\frac{1}{\psi} - 1 \right) \right) \right).$$

To show that $D_l < 0$ we first consider the case when $\psi \leq 1$. By inspection of equation (15) we observe that $D_l < 0$ if $\psi \left(1 + \sigma_l \left(\frac{1}{\psi} - 1 \right) \right) < 1$, which is true. In the case when $\psi > 1$, we rewrite D_l using equation (11) to obtain:

$$D_l = -\frac{1}{\theta_l^F \theta_l^I} y_l \frac{(1-\gamma)}{1-\sigma_l} \frac{1}{\sigma_l} \left(y_l - k \frac{\theta_l^I}{\sigma_l^{1-\gamma}} \left(1 + \sigma_l \left(\frac{1}{\psi} - 1 \right) \right) \right).$$

and then use equation (16) to show that a sufficient condition for a negative sign is that $\left(1 + \sigma_l \left(\frac{1}{\psi} - 1 \right) \right) \leq 1$, which is true. Hence $D_l < 0$ for all ψ , which ensures that $d\frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}/ds_l < 0$.

Furthermore, differentiating equations (15),(16) and (11) with respect to y_l gives the following

$$\frac{d\theta^F}{dy_l} = \frac{\theta^F}{y_l} > 0, \quad \frac{d\theta^I}{dy_l} = \frac{\theta^I}{y_l} > 0, \quad \frac{d\sigma}{dy_l} = \frac{\frac{y_l}{\theta_l^F \theta_l^I} - \frac{1}{\theta_l^I} \frac{1}{\theta_l^F} y_l}{D_l} = 0,$$

whereby $d\frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}/dy_l > 0$. Hence, we can conclude

$$\frac{\theta_h^F}{(1-\sigma_h)^{1-\gamma}} > \frac{\theta_m^F}{(1-\sigma_m)^{1-\gamma}}.$$

Thus, at least some high ability workers will educate themselves, that is, there exist an $\hat{e} > 0$ as $c(1) = 1$. Moreover, as individuals with very a low ability face very high costs of education, i.e., $\lim_{e \rightarrow 0} c(e) = \infty$, this ensures that $\hat{e} < 1$.

6.2 Higher employment rate facing highly educated workers

High productivity workers face a lower separation rate and identical or higher productivity than manual workers, that is, $s_m > s_h$ and $y_h \geq y_m$. Therefore, in order to show the difference between high productivity workers and manual workers, we consider the impact of a higher s_l and a higher y_l . Differentiating equations (15), (16) and (11) with respect to s_l gives around the equilibrium:

$$\begin{aligned}\frac{d\theta^F}{ds_l} &= -\frac{2k}{\theta_l^I} \frac{1-\gamma}{(1-\sigma_l)\sigma_l} \frac{y_l \left(\theta_l^F\right)^{\frac{1}{2}} - k \frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \left(\left(\theta_l^F\right)^{\frac{1}{2}} (1-\sigma_l) \psi + \left(\theta_l^I\right)^{\frac{1}{2}} \sigma_l \right)}{-D_l}, \\ \frac{d\theta^I}{ds_l} &= -\frac{2k}{\theta_l^F} \frac{1-\gamma}{(1-\sigma_l)\sigma_l} \frac{y_l \left(\theta_l^I\right)^{\frac{1}{2}} - k \frac{\theta_l^I}{(1-\sigma_l)^{1-\gamma}} \left(\left(\theta_l^F\right)^{\frac{1}{2}} (1-\sigma_l) \psi + \left(\theta_l^I\right)^{\frac{1}{2}} \sigma_l \right)}{-D_l}, \\ \frac{d\sigma}{ds_l} &= 2 \frac{y_l}{\theta_l^F \theta_l^I} k \frac{\left(\theta_l^F\right)^{\frac{1}{2}} - \left(\theta_l^I\right)^{\frac{1}{2}}}{-D_l} > 0,\end{aligned}$$

We can show that $\frac{d\theta^F}{ds_l}$ is negative for all ψ in the following way. Consider the case when $\psi \leq 1$. Using equation (15) and $\theta_l^F > \theta_l^I$ we observe that $\frac{d\theta^F}{ds_l}$ is negative. Next, consider the case when $\psi > 1$. Rewriting using equation (11) we obtain:

$$\frac{d\theta^F}{ds_l} = -\frac{2k}{\theta_l^I} \frac{1-\gamma}{(1-\sigma_l)\sigma_l} \frac{y_l \left(\theta_l^F\right)^{\frac{1}{2}} - k \frac{\theta_l^I}{\sigma_l^{1-\gamma}} \left(\left(\theta_l^F\right)^{\frac{1}{2}} (1-\sigma_l) + \frac{1}{\psi} \left(\theta_l^I\right)^{\frac{1}{2}} \sigma_l \right)}{-D_l},$$

which by use of equation (16) can be proved to be negative for $\psi > 1$. Hence $\frac{d\theta^F}{ds_l} < 0$ for all ψ .

The sign of $\frac{d\theta^I}{ds_l}$ is indeterminate. In case $\frac{d\theta^I}{ds_l} > 0$, then relatively labour market tightness, θ_l^F/θ_l^I , decreases with the separation rate. This in turn implies that relative employment decreases with the separation rate, $d\frac{n_l^F}{n_l^I}/ds_l < 0$. In case $\frac{d\theta^I}{ds_l} < 0$, then the negative impact on the formal sector labour market tightness is larger than the negative impact on the informal sector labour market tightness:

$$\left| \frac{\frac{d\theta^F}{ds_l}}{\frac{d\theta^I}{ds_l}} \right| = \frac{\theta_l^F y_l \left(\theta_l^F\right)^{\frac{1}{2}} - k \theta_l^I \sigma_l^{\gamma-1} \left(\left(\theta_l^F\right)^{\frac{1}{2}} (1-\sigma_l) + \left(\theta_l^I\right)^{\frac{1}{2}} \frac{1}{\psi} \sigma_l \right)}{\theta_l^I y_l \left(\theta_l^I\right)^{\frac{1}{2}} - k \theta_l^I \sigma_l^{\gamma-1} \left(\left(\theta_l^F\right)^{\frac{1}{2}} (1-\sigma_l) + \left(\theta_l^I\right)^{\frac{1}{2}} \frac{1}{\psi} \sigma_l \right)} > 0.$$

Hence also in this case we have that $d\frac{n_l^F}{n_l^I}/ds_l < 0$, which therefore holds in general for the model.

Next, as $\frac{d\theta^F}{dy_l} = \frac{\theta^F}{y_l} > 0$, $\frac{d\theta^I}{dy_l} = \frac{\theta^I}{y_l} > 0$ we obtain that $\frac{d\theta^F}{dy_l} > \frac{d\theta^I}{dy_l}$ and therefore $d\frac{n_l^F}{n_l^I}/dy_l > 0$. We can conclude that relative employment, formal relatively to informal sector, is higher for highly educated workers than for manual workers, $\frac{n_h^F}{n_h^I} > \frac{n_l^F}{n_l^I}$.

Finally, we compare unemployment rates for highly educated and manual workers. The total impact on unemployment from a higher separation rate is

$$\frac{du_l}{ds_l} = -\frac{u_l}{1 + \frac{\lambda_l^I}{s_l+p} + \frac{\lambda_l^F}{s_l}} \left(\frac{1}{s_l+p} \frac{\partial \lambda_l^I}{\partial \theta_l^I} \frac{d\theta^I}{ds_l} - \frac{\lambda_l^I}{(s_l+p)^2} + \frac{1}{s_l} \frac{\partial \lambda_l^F}{\partial \theta_l^F} \frac{d\theta^F}{ds_l} - \frac{\lambda_l^F}{s_l^2} \right).$$

As $\frac{d\theta^F}{ds_l} < 0$ then for $\frac{d\theta^I}{ds_l} < 0$ we obtain that $\frac{du_l}{ds_l} > 0$. If $\frac{d\theta^I}{ds_l} > 0$ then a sufficient condition for a positive sign is

$$\left| \frac{1}{s_l} \frac{\partial \lambda_l^F}{\partial \theta_l^F} \frac{d\theta^F}{ds_l} \right| > \left| \frac{1}{s_l+p} \frac{\partial \lambda_l^I}{\partial \theta_l^I} \frac{d\theta^I}{ds_l} \right|.$$

if and only if

$$\begin{aligned} & \frac{s_l+p}{s_l} \lambda_l^F \left(y_l (\theta_l^F)^{\frac{1}{2}} - k \frac{\theta_l^I}{\sigma_l^{1-\gamma}} \left((\theta_l^F)^{\frac{1}{2}} (1-\sigma_l) + \sigma_l (\theta_l^I)^{\frac{1}{2}} \frac{1}{\psi} \right) \right) \\ & > \lambda_l^I \left((\theta_l^I)^{\frac{1}{2}} y_l - k \frac{\theta_l^I}{\sigma_l^{1-\gamma}} \left((\theta_l^F)^{\frac{1}{2}} (1-\sigma_l) + \sigma_l (\theta_l^I)^{\frac{1}{2}} \frac{1}{\psi} \right) \right) \end{aligned}$$

which is true. Hence, $\frac{du_l}{ds_l}$ is positive.

As $\frac{d\theta^F}{dy_l} = \frac{\theta^F}{y_l} > 0$, $\frac{d\theta^I}{dy_l} = \frac{\theta^I}{y_l} > 0$ and $\frac{d\sigma_l}{dy_l} = 0$, this corresponds to $\frac{du_l}{dy_l} < 0$. Hence unemployment increases with s_l and falls with y_l . We can conclude that $u_h < u_l$.